[This question paper contains 8 printed pages.]

(13) **Your Roll No.** 2023

Sr. No. of Question Paper : 5006

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any two parts from each question.
4. **All** questions carry equal marks.

1. (a) Define limit point of a set $S \subseteq \mathbb{R}$. Find the limit points of the following sets :

P.T.O.

(i) $\mathbb{N}$

(ii) $\mathbb{R}$

(b) Define closed set. Prove that the union of finit
number of closed sets is closed set.

(c) If A and B are non-empty bounded above subse
of $\mathbb{R}$ ans $C = \{x + y | x \in A, y \in B]$ then sho
that : $\text{Sup}(C) = \text{Sup}(A) + \text{Sup}(B)$.

(d) Define neighborhood of a point and an open se
Give an example of each of the following :

(i) A non-empty set which is a neighborhoo
of each of its points with the exception
one point.

(ii) A non-empty set which is neither an open set nor a closed set.

(iii) A non-empty closed set which is not an interval.

(iv) A non-empty open set which is not an interval.

2. (a) Test the continuity of function

$$f(x)=\begin{cases}\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}, & x\neq 0\\ 0, & x=0\end{cases},$$ at $x=0$.

(b) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on $[-2,2]$.

(c) Prove that the union of an arbitrary family of open sets is an open set.

P.T.O.

(d) Show that every continuous function on a closed interval is bounded.

3. (a) Show that a sequence cannot converge to more than one limit.

(b) Show that the sequence $<a_n>$ defined by :

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots\cdots + \frac{1}{n+n}, \ \forall \ n \text{ converges.}$$

(c) State Cauchy convergence criterion for sequences and hence show that the sequence $<x_n>$ defined by :

$$x_n = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots\cdots + \frac{1}{2n-1},$$

does not converge.

(d) Show that every convergent sequence is bounded but the converse is not true.

4. (a) State Leibnitz test for convergence of an alternating series : $\sum_{1}^{\infty}(-1)^{n-1}u_n \; \forall\, n$ and test the convergence and absolute convergence of the series :

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\cdots\cdots\cdots.$$

(b) Check the convergence of the following series:

$$\sum_{n=1}^{\infty}\frac{3.6.9.\cdots 3n}{7.10.13.\cdots(3n+4)}x^n \;(x>0).$$

(c) Show that the sequence $<x_n>$ defined by :

$x_1 = 1,\; x_{n+1} = \frac{3+2x_n}{2+x_n},\; n \geq 2$ is convergent. Also find $\lim_{n\to\infty} x_n$.

P.T.O.

(d) Test the convergence of the series whose n^{th} term is $\left(\sqrt{n+1}-\sqrt{n}\right)$.

5. (a) If $<a_n>$ and $<b_n>$ are sequences of real numbers such that

$\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$ then prove that :

$$\lim_{n\to\infty} (a_n b_n) = ab.$$

(b) Define Riemann integrability of a function. Show that x^2 is integrable on any interval $[0, k]$.

(c) Show that $\lim_{n\to\infty} \frac{1}{n}\left[1+2^{\frac{1}{2}}+3^{\frac{1}{3}}+\cdots\cdots+n^{\frac{1}{n}}\right]=1$.

(d) Define the sum of a convergent series. Find the sum of the following series :

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\cdots\cdots$$

6. (a) Test the convergence and absolute convergence of the series :

(i) $\sum \frac{(-1)^{n-1}}{n^2}$.

(ii) $\sum \frac{(-1)^{n-1}}{n\sqrt{n}}$.

(b) Let $<a_n>$ be a sequence defined by

$$a_1 = 1,\ a_{n+1} = \frac{(2a_n + 3)}{4},\quad \forall\ n \geq 1,$$

Prove that $<a_n>$ is bounded above and monotonically increasing. Also find $\lim_{n\to\infty} a_n$.

P.T.O.

(c) Prove that every continuous function is integra

(d) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty} \frac{\sin nx + \cos nx}{n^{3/2}} .$$